Wide-Band, Forward-Coupling Microstrip Hybrids with High Directivity

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Abstract —The common forms of microstrip hybrids are either "backward couplers," formed using parallel lines, or branch-line or rat-race hybrids. All of these tend to have degraded performance due to discontinuities, junction effects, or unequal even- and odd-mode velocities. In contrast, coupled-line microstrip forward couplers do not require any discontinuities or junctions and utilize the unequal mode velocities. As a result, forward couplers can tolerate unusually thick substrates and still achieve high directivity and very little radiation. Though they are relatively long, designs with sizable coupling gaps have reasonable lengths for many applications, particularly at millimeter-wave frequencies. A trial symmetrical design yielded a bandwidth of 15 percent for 1-dB maximum unbalance. By use of asymmetrical design, a bandwidth of 57 percent was achieved for this same tolerance. Either quadrature or "magic-T" hybrid performance can be approximated. Directivities of 37 dB or more were readily achieved.

I. Introduction

THERE EXISTS a broad variety of 3-dB directional couplers (hybrids) that have been proposed and are used in microstrip form. The common types are branch-line hybrids and parallel-line "backward" couplers. The design theory of branch-line couplers is well understood [1, ch. 13] and application to microstrip is fairly straightforward. Conventional parallel-line coupler theory, too, is well documented [2] and has been applied to microstrip lines. A particular problem in realizing 3-dB backward couplers in parallel-coupled microstrip form is that a very narrow gap between the lines must be used. A widely used solution to this problem is the so-called Lange coupler configuration [3].

The performance of these common types of microstrip hybrids departs from their idealized responses for two major reasons. Junction effects (and bond wires in Lange couplers) introduce effects that the design theories do not account for and, in particular, degrade the directivity. These effects tend to become more pronounced as microstrip lines are being applied at higher frequencies into the millimeter-wave range. For electrically thick substrates, radiation losses at discontinuities may also become significant. Furthermore, in conventional parallel-line backward couplers, the non-TEM nature of microstrip wave propagation presents an additional problem due to the differing odd- and even-mode velocities creating unwanted forward coupling. To get higher directivity, means such as dielectric overlays are sometimes used to equalize the mode velocities. However, these can be cumbersome in practice and even then the directivity may not be high enough for some demanding applications.

In applications of coupled microstrip lines, the differing odd- and even-mode velocities are generally considered to be a nuisance which have to be "equalized" somehow while use is made of the differing odd- and even-mode impedances. Here we use an exactly opposite point of view.[1] This results in couplers where the coupling is codirectional, or forward-coupling. The basic principles of operation of these couplers are by no means new and should be quite obvious to anyone knowledgeable in directional-coupler design. However, in microstrip the use of forward coupling is unusual since it is generally believed to be very weak. Our theoretical and experimental investigations revealed, however, that forward-coupling microstrip couplers are very promising for higher frequency applications (where their relatively long electrical length can be easily accommodated) because they offer several advantages over the other conventional microstrip coupler configurations. First, they are well suited for use on relatively thick substrates because there are no abrupt discontinuities to cause junction effects and radiation. Second, with appropriately designed input and output lines, both the even and odd modes see matched loads, and this along with the lack of discontinuities virtually eliminates backward coupling so very high directivities are obtainable. Third, they have relaxed fabrication tolerances because a wide gap can be used between the strips and the resulting structure is entirely planar; i.e., no bond wires or additional dielectric layers are required. It appears that these useful properties of microstrip forward couplers have not been generally recognized by circuit designers.

II. Operating Principles

A. Symmetrical Couplers

Consider the microstrip line pattern shown in Fig. 1. Because of the symmetry, this circuit can be conveniently analyzed using the method of odd and even modes [4]. If a

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[1] It has been called to the attention of the authors that there was a previous paper published, probably around 1965 to 1975, which discusses symmetrical microstrip forward couplers. However, the specific reference has not been located.

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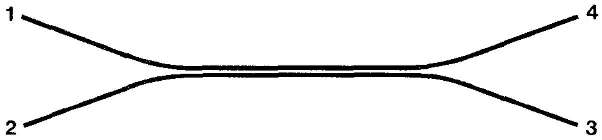

Fig. 1. The strip pattern of a symmetrical 3-dB microstrip coupler.

wave of unit voltage amplitude is incident on port 1, this excitation can be represented by two sets of one-half-volt sources connected to ports 1 and 2. For one set, the sources are in phase (even mode) and for the other they are out of phase (odd mode). The two voltage sets add up to one at port 1 and cancel at port 2. Each mode can be analyzed separately by placing an electric (odd mode) or magnetic (even mode) wall through the plane of symmetry. The effect of these walls is to alter the characteristic impedance and phase velocity of the strip where the strip is near the wall. As is well known [2], [4], the so-called backward coupling (say, from port 1 to port 2 in this structure) arises from the difference between the odd- and even-mode characteristic impedances of the coupled lines. The backward coupling is directly proportional to the difference between the reflection coefficients seen looking into a port of the structure under odd- and even-mode excitation conditions, respectively. The so-called forward coupling (say, from port 1 to port 3 in Fig. 1), on the other hand, arises from the difference between the odd- and even-mode phase velocities of the coupled lines [4]. The forward coupling is directly proportional to the difference between the complex transmission coefficients from one port to another under odd- and even-mode excitation conditions, respectively.

The structure in Fig. 1 has high directivity because it is well matched in both modes. Though the line impedances for the odd and even modes will be appreciably different where the lines are closely spaced, the terminating impedances for both modes will appear to be very well matched because of the tapered spacing between the lines towards the ends. Since the odd and even modes see negligible reflections at the ends of the lines, the net effect is as though the odd- and even-mode impedances were equal and matched to the terminations. Thus, there is very little backward coupling. However, because the odd mode is always faster than the even mode, a phase difference keeps building up between the transmitted odd- and even-mode waves, and the modes no longer cancel on the coupled line at the forward output port.

Further insight may be gained by considering a simple, uniformly coupled segment of length L. For a complete analysis, one should use equations which take the effects of both forward and backward coupling into account simultaneously, such as those in [5] (as we have done when computing frequency responses for the actual coupler structures). However, for loose couplings, we have found it a good approximation to simply separate the forward and backward coupling effects and use corresponding simplified equations in order to get a clear picture of their respective behavior. Forward coupling is expressed (neglecting that the lines have differing impedances for the odd and even modes) as

$$S_{31} = -j\sin\left(\frac{\pi \Delta n_{\text{eff}} L}{c} f\right) \tag{1}$$

where f is the frequency, c is the velocity of light in vacuum, and Δn_{eff} is defined as the difference between the square roots of the effective dielectric constants of the modes:

$$\Delta n_{\text{eff}} = \sqrt{\epsilon_{\text{eff}}^{\text{even}}} - \sqrt{\epsilon_{\text{eff}}^{\text{odd}}}. \tag{2}$$

Equation (1) displays one well-known characteristic of forward coupling: S_{31} can theoretically take on any value, even up to 0 dB, if the coupling length L is long enough, no matter how loose the coupling (i.e., small Δn_{eff}). Maximum backward coupling, however, is rapidly diminished for loose couplings, as is well known. For microstriplines, Δn_{eff} is not a strong function of frequency, and the frequency behavior of forward coupling is for the most part governed by the sine function. It can then be verified from (1) that for 3-dB coupling we may expect approximately 15 percent bandwidth for 1-dB amplitude balance tolerance, regardless of the spacing between the lines and the corresponding value of L. We may choose to use a relatively large gap between the lines, which limits backward coupling to low levels, and still get 3-dB coupling (with 15 percent bandwidth) by making L large enough. In the actual structure, backward coupling is further reduced by the impedance-matching action of the tapered spacing between the lines. In our experimental couplers, we have used simple arc-of-a-circle geometries in these sections but it is conceivable that an optimally tapered spacing might yield even better results while the overall structure could be smaller (because a narrower gap could be used between the lines and the required L would consequently be smaller).

B. Asymmetrical Couplers

Simple symmetrical forward couplers give only relatively narrow bandwidths for 3-dB coupling. It has been long recognized that asymmetrical couplers are theoretically capable of substantially wider bandwidths [6]. The asymmetry here refers specifically to a difference between the *propagation constants* of the two guides when both of them are isolated, i.e., uncoupled. According to the classic theory [6], if the uncoupled propagation constants of the two guides differ, the power traveling in one guide can be coupled only partially to the other, while full power transfer is possible between two identical guides in a symmetrical coupler. For 3-dB coupling, one might choose the parameters so that at maximum half of the power becomes coupled. Then the coupled power versus *coupler length* is flat (has zero first derivative). For transmission lines like microstrip (which is only weakly dispersive), flat coupling versus length would mean flat coupling versus frequency as well. However, even though the phase velocity of microstrip line does vary as the strip width is varied, the variation is weak. Studies of asymmetrical coupled micro-

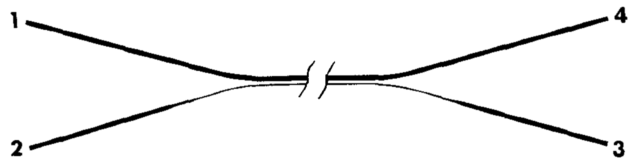

Fig. 2. The strip pattern of an asymmetrical 3-dB microstrip coupler.

strip lines revealed, however, that a large enough velocity difference can be achieved so that it is feasible to broadband a microstrip coupler by coupling two strips, one narrow and one wide. The fact that they have different impedance levels has no effect on the asymmetrical forward-coupling principle.

Asymmetrical couplers using a smoothly tapered line spacing, such as shown in Fig. 2, give high directivities. The high directivity can again be related to the differing mode impedances being impedance matched, analogously with the symmetrical case. It is well known that a completely symmetrical, lossless four-port is an ideal coupler if it is matched at all four ports [4]. A similar result can be derived for a lossless four-port which has only end-to-end symmetry, such as in Fig. 2 (see the Appendix). The coupler in Fig. 2 has also an impedance taper between the narrow line impedance and the impedance of the lines connecting the coupler to the input and output ports. However, that taper is independent of the tapered line spacing giving the high directivity. It could be omitted if the narrow line were connected to terminating impedances corresponding to its characteristic impedance. The asymmetrical coupler could therefore be used to realize an impedance transformation in addition to coupling [15].

III. Design of Asymmetrical Microstrip Couplers

Asymmetrical coupled microstrips can be conveniently analyzed in terms of the normal modes of the coupled lines [5]. For this analysis, some relevant parameters of these c and π modes must be determined: two impedances Z_{c1} and $Z_{\pi 1}$, i.e., the impedances of one line for both modes, two propagation constants β_c and β_π, and two ratios R_c and R_π, giving the ratio of the voltage on one line to the other for both modes, respectively. The other two impedances that can be defined, impedances Z_{c2} and $Z_{\pi 2}$ of the other line, can be expressed in terms of the other variables above; i.e., there are only six fully independent parameters [5]. The necessary theoretical tools to find these parameters, either in the quasi-static limit or as a function of frequency, exist [7], [8]. Often, however, microstrip circuits are designed using simplified formulas to avoid programming complicated field-theoretical solutions and lengthy calculations.

As a simple means for finding the normal-mode parameters, an empirical relation has been proposed [9]. It is based on a coupled-mode formulation of coupled microstrips [10]. For this method one needs to know the (self) characteristic impedances and propagation constants of the lines "in the presence of the other line" (or, equivalently, the self-capacitances and -inductances per unit length) and two coupling coefficients, k_C and k_L, the capacitive and the inductive coupling coefficient, respectively. The coupling coefficients are defined in terms of the self-capacitances C_i and the self-inductances L_i of the lines and their mutual capacitance C_m and mutual inductance L_m per unit length as [10]

$$k_C = C_m/\sqrt{C_1 C_2} \tag{3a}$$

$$k_L = L_m/\sqrt{L_1 L_2}. \tag{3b}$$

The relation from [9] suggests that the coupling coefficients have an exponential dependence on the separation between the lines and their widths. Suppose the widths of the strips are W_1 and W_2, their separation is S, and the substrate thickness is H. The coupling coefficients should approximately be given by

$$\log k = A - B\left(\frac{S}{H}\right) - C\left(\frac{W_1}{H} + \frac{W_2}{H}\right) \tag{4}$$

where A, B, and C are constants (different constants for the capacitive or the inductive coupling coefficient, respectively). Some values for these constants that gave a good overall agreement between this simple formula and more accurate theoretical methods were proposed in [9]. However, those constants were applicable only to relative dielectric constants greater than 6. Dispersion was not considered in [9].

We have extended this approximation by using these ideas in combination with the available fairly accurate design equations for symmetrical coupled microstrips [11]. It is believed that the overall accuracy is also thereby improved. We proceed as follows. First, the odd- and even-mode impedances and effective dielectric constants for the case of symmetrical coupled strips of width $(W_1 + W_2)/2$ separated by S are computed. On the other hand, the odd- and even-mode impedances and propagation constants of symmetrical coupled lines can be written in terms of the self and mutual capacitances and inductances as

$$Z_{e,o} = \sqrt{(L_1 \pm L_m)/(C_1 \mp C_m)} \tag{5a}$$

and

$$\beta_{e,o} = \omega\sqrt{(L_1 \pm L_m)(C_1 \mp C_m)} \tag{5b}$$

which can be obtained, for example, from [5] by making $L_1 = L_2$ and $C_1 = C_2$. (The upper signs are for the even mode and the lower signs are for the odd mode.) These can be arranged to give

$$L_1 = \frac{1}{2\omega}(\beta_e Z_e + \beta_o Z_o) \tag{6a}$$

$$C_1 = \frac{1}{2\omega}(\beta_o/Z_o + \beta_e/Z_e) \tag{6b}$$

$$L_m = \frac{1}{2\omega}(\beta_e Z_e - \beta_o Z_o) \tag{6c}$$

$$C_m = \frac{1}{2\omega}(\beta_o/Z_o - \beta_e/Z_e) \tag{6d}$$

which in turn may be used in (3a) and (3b) to find the exact coupling coefficients for this symmetrical case. It can be seen that to the extent that (4) is true, these are also the "correct" coupling coefficients for the asymmetrical case.

Besides the coupling coefficients, we need the self-inductances and -capacitances per unit length of each line or, equivalently, the self-impedances and propagation constants. The impedances and propagation constants of isolated, uncoupled lines might be used instead of their values "in the presence of the other line," as was done in [9]. However, in order to get a better approximation, one might first compute impedances and effective dielectric constants for two symmetrical cases: coupled strips of width W_1 separated by S and also strips of width W_2 separated by S. From these two sets of odd- and even-mode impedances and effective dielectric constants, one can find the self-inductances and -capacitances of the lines per unit length for the two symmetrical cases using (6a) and (6b). For an approximate solution, we can assume that the self-inductance and -capacitance of each line in the asymmetrical case remain unaltered from that in the corresponding symmetrical case.

Since we now approximately know the self-inductances and -capacitances as well as the mutual inductances and capacitances (through the coupling coefficients and (3)), we may use the equations from [5] to find the parameters of the c and π modes. This analysis is probably best for loose couplings. Dispersion can be included so that all the values that are computed for the symmetrical coupled lines are computed using frequency-dependent models, such as those in [11].

The analysis in [5] is formulated in terms of the open-circuit impedance matrix of the coupled-line four-port. From such a formulation, it is not readily apparent how the parameters of the lines should be fixed so as to get wide-band 3-dB coupling as explained before, though the formulation contains all the necessary information to compute frequency responses for any arbitrary terminating impedances. Since backward coupling is going to be weak in the structure, it may be neglected altogether if the purpose is just to fix parameters for 3-dB forward coupling. If it is further assumed that the structure is matched at all ports, the situation becomes completely equivalent to that of [12] and the mode-superposition analysis developed there can be applied. From (6) and (7) of [12] it can be seen that for a coupler with 3-dB maximum coupling versus frequency (as discussed in Section II), we need

$$-\frac{R_c}{R_\pi} = 3 \pm \sqrt{8} \tag{7}$$

and that, for a uniform coupler, the coupler length must be

$$L = \frac{\pi}{\beta_c - \beta_\pi}. \tag{8}$$

We have found in practice that (7) and (8) give excellent agreement with complete computations where the differing mode impedances are taken into account.

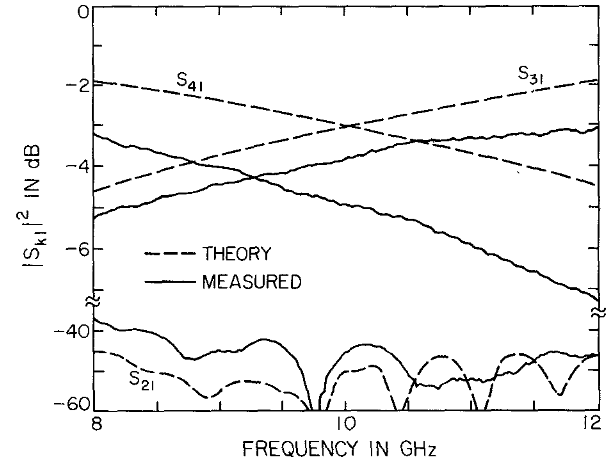

Fig. 3. Theoretical and experimental results for the symmetrical coupler shown in Fig. 1.

The analysis above assumes uniform lines, whereas the actual structure has coupled lines of varying spacing. The curved, coupled strips of these input and output lines have been analyzed by dividing them into small segments which are then analyzed as being parallel and uniformly coupled. The separation between these elements has been measured along an arc of a circle intersecting the strips at right angles. This should be a good approximation since a relatively large radius of curvature must be used to ensure proper impedance-matching action in these sections.

IV. Experimental Results

A. Symmetrical Coupler

An experimental hybrid was designed to be operated at 10 GHz. The substrate material was Duroid with a dielectric constant of 2.20 and a thickness of 0.762 mm. The strip pattern of the coupler is shown in Fig. 1. The line width, which is constant throughout, corresponds to 50 Ω for an isolated line. The length of the straight middle part is 113 mm, or about 5.2 guided wavelengths at 10 GHz. The strip spacing there is equal to twice the substrate thickness. The curved strips have a radius of curvature of 102 mm. A cascade of 20 segments, each 1.27 mm ($=0.059\lambda_g$) long, was used to represent them in the computations. The coupled microstrip lines were analyzed using formulas from [11]. A computed frequency response for the whole structure is shown by the dashed lines in Fig. 3. The importance of the tapered strip spacing can be understood if we consider that the theoretical minimum directivity for a uniform coupler with no tapered spacing and 50-Ω loads would be about 24 dB while we predict directivities on the order of 40 dB for the actual structure.

Corresponding measured results obtained with an HP 8510 network analyzer are shown by the solid lines in Fig. 3. The shape of the measured S_{41} and S_{31} agree well with the theoretically calculated curves but the center frequency is low. The reason for this could be the limited accuracy with which the odd- and even-mode velocities are known. Note that even if these velocities are known, say, to, within a few percent, the percent error in their *difference* can be large. The measured insertion loss of the coupler, which is about 1.3 dB, includes losses in the fairly long input and output lines (see Fig. 1) and the coax-to-microstrip transitions. The phase difference between S_{41} and S_{31} is known to be fixed at 90° due to the symmetry of the coupler,[2] and measurements showed the outputs to be in quadrature within less than 5°. Part of the measured deviation from 90° could also easily be due to effects such as different electrical lengths of nominally identical connectors.

Directivity was measured by carefully minimizing reflections from ports 3 and 4 and measuring the isolation between ports 1 and 2. To do this, we connected loads to the coaxial connectors on ports 3 and 4, and added gradually tapered absorbing material next to the lines leading to these two ports in order to attenuate any residual reflections from the coaxial loads and the coax-to-microstrip transitions (this is why we had such long input and output lines). The measured isolation S_{21} is close to the theoretically predicted level of about 40 dB over most of the band.

B. Asymmetrical Coupler

An asymmetrical 3-dB coupler was also designed and tested. The strip pattern of this coupler is shown in Fig. 2. The same Duroid substrate material as for the symmetrical coupler was used. The widths of the lines were chosen to correspond to (uncoupled) impedances of 50 and 100 Ω. For this substrate and these line widths, and using (7) and our approximate method of analysis, it was found that the separation between the lines should be 1.81 mm to give 3-dB maximum coupling. According to (8), the length of a uniform coupler should then be 220 mm, or about 10 guided wavelengths, to give this maximum at 10 GHz. In the actual experimental coupler, a spacing of 1.65 mm was used instead, which allowed more than 3-dB coupling at the center frequency but gave wider bandwidth for 1-dB amplitude balance, which was used as a design goal (see the dashed lines in Fig. 4). Also, the length of the actual coupler was corrected to compensate for coupling in the curved sections (as well as for the narrower gap). In the final design, the length of the straight center part was 190 mm with the curved sections having a radius of curvature of 102 mm. Frequency responses were initially computed assuming 50-Ω terminations for the wide strip and 100-Ω terminations for the narrow strip. The coupler had a 54-percent bandwidth centered at 9.6 GHz and better than 40-dB isolation over this band.

Since the coupler was to be operated with 50-Ω terminations, an impedance transformer was needed between the 100-Ω strip and the 50-Ω terminating strips. Note that this transformer must have a return loss better than the isolation of the coupler if it is not to degrade the directivity. A taper was designed using Hecken's near-optimum function [13] for 50-dB return loss. In the actual implementation of the taper, the width was fixed at ten points and a linear variation of width was used in between. The variation of the propagation constant of microstrip as a function of width was ignored in this design. The total length of the taper is 40 mm, which theory says should give a good match for frequencies of 5.3 GHz and above.

[2] apart from the effects of any differential mode loss.

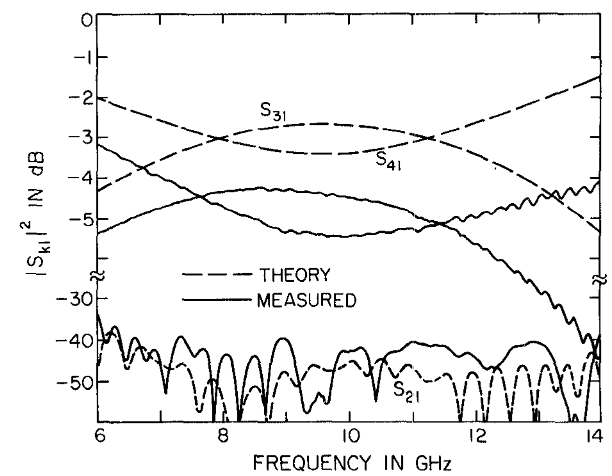

Fig. 4. Theoretical and experimental results for the asymmetrical coupler shown in Fig. 2

A computed frequency response for the complete structure is shown by the dashed lines in Fig. 4. Corresponding measured results are shown by the solid lines. The measured bandwidth for 1-dB unbalance is 57 percent centered at 9.5 GHz. Agreement with theory is quite good, taking into account that the theoretical calculations ignored losses. The measurement in Fig. 4 includes all the losses in the long input and output lines of the coupler as well as in the coax-to-microstrip transitions, and their contributions is believed to be a significant part of the measured loss. Beforehand we had anticipated possible differences in the through-line frequency characteristics depending on whether the wide or the narrow line is driven due to possible differential mode loss, but such effects were found to give differences of less than a few tenths of a dB.

Isolation was measured in a similar manner as for the symmetrical coupler. The measured isolation exceeding 40 dB over most of the band verifies that the coupler has a very high directivity *and* that the 100-to-50-Ω impedance tapers meet their design goals quite successfully.

Theoretically calculated and measured phase differences between the "coupled" and "through" ports for the two possible excitations are shown in Fig. 5. As is well known and can be seen from Fig. 5, asymmetrical couplers do not possess inherent constant-phase-difference properties between their output ports as symmetrical couplers do [4]. However, it turns out that it is still possible to approximate desirable phase-difference responses simply by using a length of line as a phase-compensating element. Either quadrature or "magic-T" performance can be obtained. To get the 90° case, one could add a length of line having a 106° phase shift at 9.6 GHz to ports 2 and 3, making the outputs to be in quadrature at that frequency. Calculated phase differences between the outputs with this fixed length of line in place are shown with the solid lines in Fig.

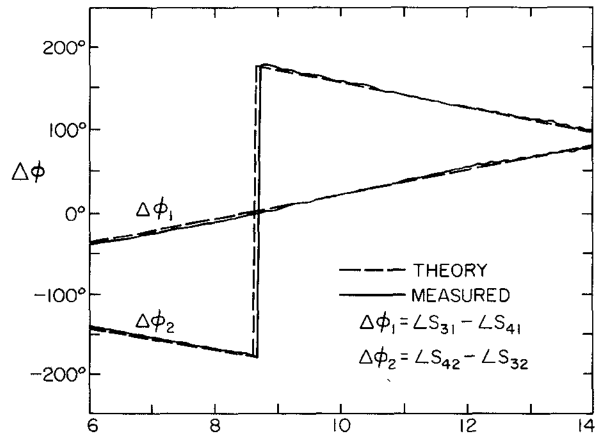

Fig. 5. Theoretically computed and measured phase differences between the coupled and through ports of the asymmetrical coupler in Fig. 2.

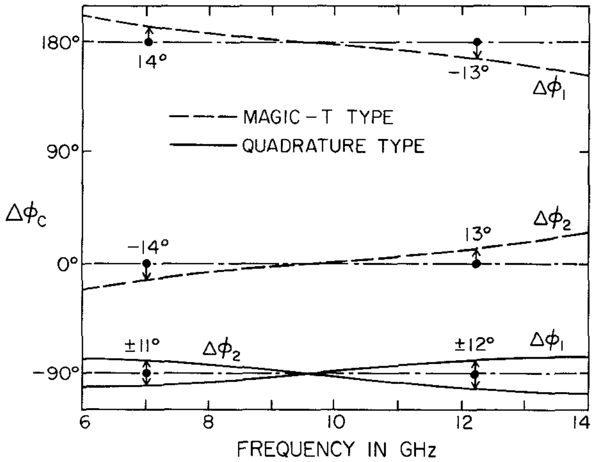

Fig. 6. Theoretically computed phase differences between the coupled and through ports of the asymmetrical coupler with reference planes chosen to approximate quadrature or magic-T performance $\Delta\phi_1$ and $\Delta\phi_2$ are as defined in Fig. 5.

6. The outputs are held in quadrature with about 12° tolerance over the 1-dB amplitude balance band (7.0 to 12.2 GHz). Using a line still longer by a quarter wavelength at 9.6 GHz, the outputs would be in phase when port 2 is driven, while they would be out of phase for excitation at port 1 due to the phase-difference properties of these couplers (see the Appendix). A tolerance of about 14° could be maintained for this magic-T case.

V. Conclusions

The odd- and even-mode velocities of coupled microstrip lines are sufficiently different to permit 3-dB forward coupling with reasonable length for millimeter-wave applications. These couplers easily achieve directivities on the order of 40 dB because the so-called backward coupling due to differing odd- and even-mode impedances can be eliminated in a suitable structure which has no abrupt discontinuities.

Symmetrical couplers are the simplest structures of this class. They give about 15 percent bandwidths for 1-dB unbalance centered about 3-dB coupling. Asymmetrical hybrids, on the other hand, can approach octave bandwidths for the same tolerance. The reference planes of an asymmetrical hybrid can be chosen to give approximately quadrature or magic-T-type characteristics. These couplers are simple to design and implement, even at very high frequencies. They should be useful for applications demanding high directivities or for millimeter-wave applications.

Appendix
Properties of Lossless Asymmetrical Couplers

Lossless, asymmetrical backward couplers have been shown to have infinite directivity if they are matched at all four-ports [14] and [15].[3] It has also been observed that the phase differences between the outputs of an asymmetrical backward coupler obey a certain relation [16]. It turns out these results are valid not only for backward couplers, but are properties of a certain class of lossless four-ports. A proof is obtained here in a general framework with no restrictions as to the type of four-port (i.e., backward coupler, forward coupler, or other).

Consider a four-port for which two of its ports are electrically indistinguishable from each other, as are the remaining two. The first pair of ports, however, may be distinguishable from the other pair. The coupler in Fig. 2 is an example of such a network. Ports 1 and 4 are alike, as are 2 and 3, and it can be argued from symmetry that, e.g., $S_{31} = S_{24}$. Assuming reciprocity and complete matching at all four ports, the scattering matrix for the coupler in Fig. 2 can be reduced to

$$\boldsymbol{S} = \begin{bmatrix} 0 & S_{21} & S_{31} & S_{41} \\ S_{21} & 0 & S_{32} & S_{31} \\ S_{31} & S_{32} & 0 & S_{21} \\ S_{41} & S_{31} & S_{21} & 0 \end{bmatrix}. \tag{A1}$$

$\boldsymbol{S}$ is known to be unitary for lossless networks, i.e.,

$$\boldsymbol{S}\boldsymbol{S}_t^* = \boldsymbol{I} \tag{A2}$$

where $\boldsymbol{S}_t^*$ refers to the matrix obtained from $\boldsymbol{S}$ through transposing and taking the complex conjugate, and $\boldsymbol{I}$ is the unit matrix. Condition (A2) leads to equations

$$S_{31}S_{32}^* + S_{41}S_{31}^* = 0 \tag{A3}$$

$$S_{21}S_{32}^* + S_{41}S_{21}^* = 0 \tag{A4}$$

$$S_{21}S_{31}^* + S_{31}S_{21}^* = 0. \tag{A5}$$

The superscript * refers to the complex conjugate.

An Isolation Property: Multiply (A4) by S_{31}, substitute for $S_{31}S_{32}^*$ from (A3), and use (A5) to express $S_{21}S_{31}^*$ to derive the result

$$2S_{21}^*S_{31}S_{41} = 0. \tag{A6}$$

Equation (A6) shows that if port 1 of the coupler is driven, (at least) one of the remaining three ports is isolated. For a

[3] The asymmetries that these two papers dealt with were of different types, but both kinds of networks fall into the general class of four-ports considered here with proper identification of corresponding ports.

forward coupler $S_{21}=0$. A similar result can be obtained concerning port 2 being driven. The key assumption under this conclusion is that the four-port was assumed to be matched at all four ports.

A Phase-Difference Property: Equation (A3) can be written in a different form as

$$\frac{S_{31}}{S_{41}}\left(\frac{S_{32}}{S_{31}}\right)^{*}=-1. \tag{A7}$$

If this equation is written as two real equations, one for the magnitudes of each side and one for the phase angles of each side, we get for the phase-angle part

$$\arg\left(\frac{S_{31}}{S_{41}}\right)+\arg\left(\frac{S_{31}}{S_{32}}\right)=\pi. \tag{A8}$$

In (A8) the first term on the left-hand side is the phase difference between the "coupled" and "through" ports when port 1 is driven while the second term is the phase difference between the coupled and through ports when port 2 is driven (note that $S_{31}=S_{42}$ by symmetry and reciprocity). According to (A8), they add up to 180°.

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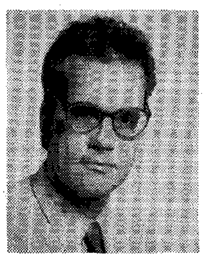

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